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Adaptive neural network control of an uncertain robotic manipulator with input constraint and external disturbance

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Abstract: This paper investigates the control problem of an uncertain robotic manipulator under the effect of bounded external disturbance and input constraint. A novel Neural Network (NN) controller is proposed to achieve asymptotic tracking of desired trajectory. The model uncertainty is lumped together with external disturbance and compensated by the NN term of the proposed adaptive controller, while the boundedness of input is ensured via an auxiliary system and a projection operator. The ISS property of the closed-loop system and the boundedness of input are rigorous proved. Finally, we show the effectiveness of the proposed controller in the numerical study.

Key Words: Robotic manipulator, Adaptive control, Neural network, Input constraint

1 Introduction

Robotic manipulator finds a wide range of applications in engineering practice, such as industrial assembly [1], underwater detection [2], and so on. The tracking problem of robotic manipulator has been extensive studied in the past decades [3, 4] due to four major difficulties encountered in practice: nonlinearity and uncertainty of system model, unknown external disturbance and input constraint.

Given accurate model, enormous control schemes have been developed to deal with unknown external disturbance, starting from earliest PID control [5]. Although the effectiveness of model-based methods [6] can be proved in some applications [7], there are many more practical scenarios where the uncertainty of the model is inevitable.

Put model uncertainty and external disturbance together (usually term as lumped uncertainty), a great amount of techniques have been extensively studied. For instance, sliding mode control disturbance observer-based on [8] enjoys the strong robustness while suffers from chattering phenomenon; [9] proposes a fuzzy parameter self-tuning controller that has both the accuracy of PI control and the adaptability of the fuzzy control, but the fuzzy control rules are assumed to be a priori known information; [10] is robust both to parametric uncertainty and to external disturbance, however the accuracy is less satisfactory due to the conservative choice of fixed parameters; this shortcoming can be overcomed by adaptive control-based method, such as [11]. Among them, the adaptive methodology [12] and neural network (NN) based mechanism [13, 14] are becoming appealing to the researchers and engineers due to their ability of on-line adjustment of controller parameters and approximation of nonlinear dynamics in a linear manner, respectively. The complexity of adaptive NN methods used to be a big concern, but now are less interested. Consequently, the NN-based adaptive control has been widely used in the control of robotic manipulator [15–18] to handle uncertainty. In [19], the NN is even used to approximate the whole dynamics of robot, thus achieve a completely model-free control mechanism. For the lumped uncertainty considered in this paper, we can approximate it by NN —more specifically, the radial basis function (RBF) NN— to any desired degree of precision [20], given sufficiently large number of neurons. Besides, the universal approximation property of NNs holds over a compact set depending on the arguments of the function under approximation, and this set can be easily determined in feedforward NN compensator due to the known and bounded arguments of the function [21]. Consider this and the nice property of adaptive feedforward control [22, 23], we therefore use an adaptive feedforward NN to compensate the lumped uncertainty as well.

Despite the high precision of aforementioned adaptive NN methods, the constraint of actuator are rarely considered, which, if not properly coped with, would degrade system performances and even cause instability [24, 25]. [21] and [26] both propose the controller with bounded input, however, the boundary of the control signal cannot be determined by the designers, that is, whether this bounded input satisfies the limits of actuator or not, is unknown. In practice, the limits of actuator are easily obtained, therefore one would like to bound the control signal by the limits before it goes to actuator. It is natural to do this by applying a sigmoid function to the original control law, as [24]. However, in this way, certain sacrifices of performance is inevitable, especially when the control signals are saturated during the whole transient period. Compared with the sigmoidal function, hyperbolic tangent function $\tanh(\cdot)$ is introduced in [27] and [28] to improve the control performance with input constraint. But, the controller in [27] will waste model informance that a priori known while the one proposed in [28] is built on the premise that the model can be accurately modelled. Furthermore, [29] proposes a bounded controller with adjustable boundary by taking the upper bound for velocity and acceleration of the given tracking trajectory into consideration, but it also requires an accurate model.

Motivated by above observations, this paper proposes a novel neural network-based adaptive control for uncertain robotic manipulator systems with input constraint and external disturbance. First, based on the nominal model, a feedback controller is designed. By defining new auxiliary dynamics, a priori bounded control command can be guaranteed. A feedforward RBF NN term is then added to com-

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pensate the model uncertainty and the external disturbance, while a projection operator is used to ensure the boundedness of NN's output. The ISS stability analysis demonstrates a clear dependence of tracking error on the boundary of input and size of uncertainty.

The main contribution of this paper is to propose a new and flexible NN-based adaptive controller that not only maximize the use of the prior knowldge of nominal model, but also able to deal with large model uncertainty and external disturbance. If the model of plant is sufficiently accurate, our method tracks the desired trajectory with high precision; in the case that the system is not well-modeled or faces significant disturbance, the proposed method demonstrates strong robustness. The switching between two scenarios is simply done via adjusting one tuning parameter η. In either case, the propose method ensures the stability of the closed-loop system. Another major novelty of the proposed controller is that, the control signals are always guaranteed to be bounded by the limits of actuator. In addition, consider desired trajectory, size of lumped uncertainty and the limitation of actuator, if we know two of the three, the rest of term can be determined accordingly.

2 Problem Formulation

In this section, we formulate the trajectory tracking problem of an n-link robotic manipulator modeled by the following Euler-Lagrange equation [30]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + T_d = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are, respectively, the vectors of angular position, velocity, and acceleration of joints, $M(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix of the whole system, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the centrifugal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ is the friction vector, $T_d \in \mathbb{R}^n$ and $\tau \in \mathbb{R}^n$ represent the external disturbance and the output torque of actuator, respectively.

In this work, a nominal model with a bounded model uncertainty is assumed to be available and are denoted by $M_o(q), C_o(q,\dot{q}), F_o(\dot{q}), G_o(q)$. The model mismatch is represented by $\Delta M(q)$, $\Delta C(q,\dot{q})$, $\Delta F(\dot{q})$, $\Delta G(q)$ and verify the following relations:

$$\begin{aligned} &M_o(q) \triangleq M(q) + \Delta M(q) \quad C_o(q,\dot{q}) \triangleq C(q,\dot{q}) + \Delta C(q,\dot{q}) \\ &F_o(\dot{q}) \triangleq F(\dot{q}) + \Delta F(\dot{q}) \quad G_o(q) \triangleq G(q) + \Delta G(q) \end{aligned} \tag{2}$$

The following properties [7] for the manipulator system are well-known and are fundamental to subsequent analysis.

Property 1 *The time derivative of inertia matrix $M(q)$ and the centrifugal-Coriolis matrix $C(q,\dot{q})$ satisfy the following skew-symmetric relationship:*

$$x^T(\dot{M}(q) - 2C(q,\dot{q}))x = 0, \quad \forall x \in \mathbb{R}^n$$

Property 2 *The positive-definite symmetric inertia matrix $M(q)$, the centrifugal-Coriolis matrix $C(q,\dot{q})$, the friction $F(\dot{q})$ and gravitational vector $G(q)$ satisfy the following inequalities:*

$$\begin{aligned} &\underline{m}||x||^2 \le x^T M(q) x \le \overline{m}||x||^2, \quad \forall x \in \mathbb{R}^n \\ &||C(q,\dot{q})|| \le \zeta_c||\dot{q}||, \quad ||F(\dot{q})|| \le \zeta_{f_d}||\dot{q}|| + \zeta_{f_s} \\ &||G(q)|| \le \zeta_g \end{aligned} \tag{3}$$

for some known constants ζ_c, ζ_{f_d}, ζ_{f_s}, $\zeta_g \in \mathbb{R}^+$, and $\underline{m}$, $\overline{m} \in \mathbb{R}^+$ are defined as $\underline{m} \triangleq \min_{\forall q\in\mathbb{R}^n} \lambda_{min}(M)$ and $\overline{m} \triangleq \max_{\forall q\in\mathbb{R}^n} \lambda_{max}(M)$, respectively.

Property 3 *The speed limits of actuator of the system is known and defined as:*

$$\beta \triangleq \max_{i\in\{1,\cdots,n\}} \sup_{t\ge0}\{\dot{q}_i(t)\} \tag{4}$$

The knowledge of external disturbance and input constraint are given in the next two assumptions.

Assumption 1 *The external disturbance term T_d is unknown but assumed to be norm-bounded by*

$$||T_d(t)|| \le \zeta_d \tag{5}$$

where $\zeta_d \in \mathbb{R}^+$ is a given positive constant. ◁

Assumption 2 *Let τ_N represents the limits of the output torque of actuator and assumed to be known prior. When there is model uncertainty, τ_N needs to be large enough to verify the following inequality*

$$\tau_N > \tau_m + \zeta_{f_s} + \zeta_g \tag{6}$$

for ζ_{f_s} and ζ_g are constants referred in the Property 2, τ_m is a positive constant defined in (52). ◁

Furthermore, with bounded input signal, the system certainly can not track an unlimited designed trajectory. It's not conservative to assume that the desired trajectory vector $q_d(t)$ is a $\mathcal{C}^2$ function vector and $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t) \in \mathcal{L}_\infty$, such that we define their upper bounds as

$$\underset{t\ge0}{sup}||\dot{q}_d(t)|| = B_v \quad \underset{t\ge0}{sup}||\ddot{q}_d(t)|| = B_a$$

where B_v and B_a are positive constants that need to proper designed. It is important to note that the upper bound for the velocity of the tracking trajectory should satisfy the following inequality

$$\max_{i\in\{1,\cdots,n\}} \sup_{t\ge0}\{\dot{q}_{di}(t)\} \le \beta \tag{7}$$

Hence, we have $B_v \le \sqrt{n}\beta$.

The trajectory tracking error signals are represented by $e(t) \triangleq q(t) - q_d(t)$ and $\dot{e}(t) \triangleq \dot{q}(t) - \dot{q}_d(t)$. Therefore, the problem considered in this paper is formally stated as follows:

Problem 1 *Given Assumptions 1-2, design a trajectory tracking controller with a bounded control law $\tau \triangleq \tau(q,\dot{q})$ for an n-link robot manipulator (1), such that $||\tau|| \le \tau_N$ and the closed-loop system is ISS with respect to disturbance and unmodeled dynamics. That is, there exist class $\mathcal{K}$ functions $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ verifying the following inequalities of error signals as*

$$\lim_{t\to\infty} ||e(t)|| \le \gamma_1(\Delta f) \quad \lim_{t\to\infty} ||\dot{e}(t)|| \le \gamma_2(\Delta f) \tag{8}$$

with Δf denotes the lumped uncertainty defined in (21). ◁

Remark 2.1 *It's worth pointing out, different from the existing methods in the literature, here we do not bound the input by a saturation function. Instead, by appropriate choice of the tunning gains, the proposal algorithm guarantees that the control input will not exceed the boundary presented in Assumption 2. In this way, the performance of the system will not degrade in pratical implementation. Furthermore, the proposed scheme provides some insights of the performance limitation of system.*

3 Controller Design

This section is devoted to the design of the NN-based adaptive controller, where two auxiliary signals are introduced to assistant design, while the NN term is used to compensate the lumped uncertainty. We start with rewriting the dynamics in (1) as

$$M(q)\ddot{e} = \tau - C(q,\dot{q})\dot{q} - F(\dot{q}) - G(q) - T_d - M(q)\ddot{q}_d \quad (9)$$

and introducing the auxilary variables $r \in \mathbb{R}^n$ and $\chi \in \mathbb{R}^n$:

$$r = \dot{e} + \frac{\alpha}{1+\beta'} \cdot \mathrm{Tanh}(e) + \chi(e) \quad (10)$$

$$\dot{\chi} = -K_1 r - K_2\chi + \mathrm{Tanh}(e), \quad \chi(0) = 0 \quad (11)$$

where $\alpha \in \mathbb{R}^+$ is a constant adjustable gain, $K_1, K_2 \in \mathbb{R}^{n\times n}$ are diagonal positive-definite gain matrices, $\beta' \in \mathbb{R}^+$ denotes the upper bound of the derivative of error, the bounded vector function $\mathrm{Tanh}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is defined as:

$$\mathrm{Tanh}(x) \triangleq [\tanh(x_1), \cdots, \tanh(x_n)]^T$$

for any n-dimensional vector x.

Note that χ and r are available signals and differentiable. Substitute the derivative of r which is defined in (10) into (9) such that we can obtain

$$M(q)\dot{r} = -C(q,\dot{q})r - M(q)K_1 r + f_1 + f_2 + \tau - T_d \quad (12)$$

where f_1 and f_2 are given by

$$f_1 = M(q)[\frac{\alpha}{1+\beta'}\mathrm{Cosh}^{-2}(e)\dot{e} - K_2\chi + \mathrm{Tanh}(e)] \quad (13)$$

$$f_2 = C(q,\dot{q})r - C(q,\dot{q})\dot{q} - F(\dot{q}) - G(q) - M(q)\ddot{q}_d \quad (14)$$

with the vector function $\mathrm{Cosh}(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is defined similar to $\mathrm{Tanh}(\cdot)$ as

$$\mathrm{Cosh}(x) \triangleq \mathrm{diag}\{\cosh(x_1), \cdots, \cosh(x_n)\} \quad (15)$$

for any n-dimensional vector x.

Given e, $\dot{e}$ and nomial models of the system, we estimate the terms f_1 and f_2 respectively by:

$$\hat{f}_1 = M_o(q_d)[\frac{\alpha}{1+\beta'}\mathrm{Cosh}^{-2}(e)\dot{e} - K_2\chi + \mathrm{Tanh}(e)] \quad (16)$$

$$\hat{f}_2 = C_o(q_d,\dot{q}_d)[-\dot{q}_d + \frac{\alpha}{1+\beta'} \cdot \mathrm{Tanh}(e) + \chi] - F_o(\dot{q}_d) - G_o(q_d) - M_o(q_d)\ddot{q}_d \quad (17)$$

Here we use q_d and $\dot{q}_d$ instead of measurements q and $\dot{q}$ since, in general, they are defined by users and hence are not prone to be augmented with noise [31].

From (13), (14), (16) and (17), with nonzero model mismatch, there exist estimation errors $\tilde{f}_1 \triangleq f_1 - \hat{f}_1$ and $\tilde{f}_2 \triangleq f_2 - \hat{f}_2$, admitting the form of

$$\tilde{f}_1 = \tilde{M}[\frac{\alpha}{1+\beta'}\mathrm{Cosh}^{-2}(e)\dot{e} - K_2\chi + \mathrm{Tanh}(e)] \quad (18)$$

$$\tilde{f}_2 = \tilde{C}[-\dot{q}_d + \frac{\alpha}{1+\beta'}\mathrm{Tanh}(e) + \chi] - \tilde{F} - \tilde{G} - \tilde{M}\ddot{q}_d \quad (19)$$

where the tilde symbol represents the error between the nominal value and true value in the form of

$$\tilde{\Sigma} = \Sigma(q,\dot{q}) - \Sigma_o(q_d,\dot{q}_d) = \Sigma(q,\dot{q}) - \Sigma(q_d,\dot{q}_d) - \Delta\Sigma(q_d,\dot{q}_d)$$

with $\Sigma \in \{M, C, F, G\}$.

According to (7) and Property 3, we have the upper bound of velocity error $\beta' := 2\beta \geq ||\dot{e}_i||$ for $i = 1, 2, ...n$. In view of Property 2 and 3, one can easily verify that $\tilde{M}$, $\tilde{C}$, $\tilde{F}$ and $\tilde{G}$ are bounded. If χ is also bounded, which will be proved in the Section 4, then the boundedness of $\hat{f}_1$, $\hat{f}_2$, $\tilde{f}_1$ and $\tilde{f}_2$ are straightforward to obtain.

Provided the estimated term, we rewrite (12) as:

$$M(q)\dot{r} = -C(q,\dot{q})\dot{r} - M(q)K_1 r + \hat{f}_1 + \hat{f}_2 + \Delta f + \tau \quad (20)$$

where Δf represents the lumped uncertainty, defined as:

$$\Delta f = \tilde{f}_1 + \tilde{f}_2 - T_d \quad (21)$$

Due to the nonlinearity of the lumped uncertainty, we use a neural network to approximate it. According to [32], Δf can be parameterized as

$$\Delta f = W^T\sigma(z) + \epsilon(z) \quad (22)$$

where $\epsilon(z) \in \mathbb{R}^n$ is the functional reconstruction error norm-bounded by a positive constant ϵ_N, and $W \in \mathbb{R}^{\mathcal{L}\times n}$ is a bounded constant ideal weight matrix with $\mathcal{L}$ denoting the number of neurons which is determined by users, $\sigma(z) \in \mathbb{R}^{\mathcal{L}}$ is the gaussian activation function defined as $\sigma(z) \triangleq [\sigma(z) \cdots \sigma_{\mathcal{L}}(z)]^T$ with the basis gaussian functions $\sigma_i(\cdot)$ which are given by $\sigma_i(z) \triangleq \prod_{k=1}^m e^{-(z_k-\mu_{ik})^2/2p_{ik}}$. Parameters z_k, μ_{ik} and p_{ik} are, respectively, the kth components of z, the mean vector $\mu_i \in \mathbb{R}^m$, and the corresponding diagonal covariance matrix $P_i = diag\{p_{ik}\} \in \mathbb{R}^{m\times m}$.

Now, we give the estimate of Δf as follows:

$$\Delta\hat{f} = \hat{W}^T\sigma(z) \quad (23)$$

where $z = [1, q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d]^T \in \mathbb{R}^{5n+1}$, $\hat{W}$ represents an estimate of the ideal weight W of NN, updated by

$$\begin{aligned} \dot{\hat{W}} &= \Gamma\dot{w} \\ \dot{w} &= \sigma(z)\dot{e}^T + \sigma(z)(\alpha e + \chi)^T - \rho\hat{W} \end{aligned} \quad (24)$$

with $\Gamma \in \mathbb{R}^{\mathcal{L}\times\mathcal{L}}$ is a constant positive-definite symmetric matrix, $\rho \in \mathbb{R}^+$ is a constant gain. Note that, (24) leads to a bounded output $\Delta\hat{f}$, but the boundary is nonadjustable. In order to bound the input τ as we want, $\hat{W}$ needs to be projected such that $\Delta\hat{f}$ remain in the desired bounded region. Therefore, we add a standard projection operation to (24) as:

$$\dot{\hat{W}} = \begin{cases} \Gamma\dot{w}, & g < 0 \text{ or } (g = 0 \text{ and } \dot{\hat{W}} \cdot \nabla g \leq 0) \\ \Gamma\dot{w} - \Gamma\dfrac{\nabla g \nabla g^T}{\nabla g^T \Gamma \nabla g}\Gamma\dot{w}, & \text{otherwise} \end{cases} \quad (25)$$

where $\nabla(\cdot)$ stands for finding the gradient of the function, and $g = \hat{W}^T \cdot \sigma(z) - \eta \cdot \tau_N$ represents the subject function. $\eta \in (0, 1)$ is a tunable parameter which represents the percentage of controller's effort that devoted to handle the uncertainty. The tuning principle of η will be given in Section 4.

Remark 3.1 *The number of neurons and the value of b_i are related to the size of approximation error ϵ_N and should be selected carefully. But the optimal design of NN is out of scope of this paper.*

Finally, for the dynamics (20), we propose the following control law:

$$\tau = K_1\chi - \hat{f}_1 - \hat{f}_2 - \varDelta\hat{f} \tag{26}$$

K_1 is a positive tunning gain matrix and variables χ, $\hat{f}_1$, $\hat{f}_2$, $\varDelta\hat{f}$ are defined in (11), (16), (17) and (23), respectively.

4 Stability and Boundness Analysis

This section first denotes to the convergence analysis of the closed-loop system consist of the plant (1) and control law (26). Then the boundness of control input is rigorous proved.

Substituting the control law (26) into (20), the closed-loop system is written in terms of r

$$M(q)\dot{r} = -C(q,\dot{q})r - M(q)K_1 r + K_1\chi + \tilde{W}^T\sigma(z) + \epsilon(z) \tag{27}$$

with $\tilde{W}(t) \in \mathbb{R}^{\mathcal{L}\times n}$, defined as $\tilde{W} \triangleq W - \hat{W}$, denotes the estimate mismatch for the ideal weight matrix. (27) features ISS as claimed in the next Theorem.

Theorem 4.1 *Suppose Assumptions 1-2 hold, the closed-loop system* (27) *is ISS if the control gains α, K_1 and K_2 satisfy the following conditions:*

$$\alpha > (1 + \beta') \tag{28}$$

$$\lambda_{min}(K_1) > \frac{1+\kappa_r^2}{4\underline{m}} \tag{29}$$

$$\lambda_{min}(K_2) > 0 \tag{30}$$

with κ_r is a positive constant defined in subsequent proof. Moreover, the tracking error e and $\dot{e}$ asymptotically converges to a small neighborhood of the origin whose size depends on tuning parameter and lumped uncertainty,

Proof 1 *Consider the following candidate lyapunov function:*

$$V(h) = \sum_{i=1}^{n} ln[\cosh(e_i)] + \frac{1}{2}r^T M r + \frac{1}{2}\chi^T\chi + Q$$

where $Q = \frac{1}{2}tr(\tilde{W}^T\varGamma^{-1}\tilde{W})$ and $h = [e^T, r^T, \chi^T, \sqrt{Q}]^T \in \mathbb{R}^{3n+1}$. Thanks to the fact that $ln[\cosh(||x||)] \leq ||x||^2$ for all $x \in \mathbb{R}^n$, it is easy to bound V from below and above as

$$\rho_1 ln(\cosh(||h||)) \leq V \leq \rho_2||h||^2 \tag{31}$$

for some positive constants ρ_1, ρ_2 defined as $\rho_1 = \min\frac{1}{2}\{1, \underline{m}\}$, $\rho_2 = \max\{1, \frac{1}{2}\overline{m}\}$.

Referring to (1)*,* (10) *and* (27)*, the system is the time derivative of $V(h)$ along the trajectory of*

$$\dot{V} = -\frac{\alpha}{1+\beta'}\mathrm{Tanh}^T(e)\mathrm{Tanh}(e) - r^T M(q)K_1 r - \chi^T K_2\chi + \mathrm{Tanh}^T(e)r + r^T\tilde{W}^T\sigma(z) + r^T\epsilon(z) - tr(\tilde{W}^T\varGamma^{-1}\dot{\hat{W}})$$

Thanks to the properties of the projection operator [21, 33, 34] and the Young's inequality, the following inequalities holds:

$$r^T\tilde{W}^T\sigma(z) - tr(\tilde{W}^T\varGamma^{-1}\dot{\hat{W}}) \leq tr(\rho\tilde{W}^T\hat{W}) \leq -\frac{\rho}{2}||\tilde{W}||_F^2 + \frac{\rho}{2}||W||_F^2$$

$$||W||_F^2 = tr(W^T W) \leq W_B, \quad W_B \in \mathbb{R}^+ \tag{32}$$

where ρ is a constant gain employed in the updating rate of RBF NN, W_B is a positive constant whose value is proportional to the lumped uncertainty.

Then, using Property 2 and (32)*, an upper bound for $\dot{V}$ can be obtained as:*

$$\dot{V} \leq -(\frac{\alpha}{1+\beta'} - 1)||\mathrm{Tanh}(e)||^2 - \lambda_{min}(K_2)||\chi||^2 - [\underline{m}\lambda_{min}(K_1) - \frac{1}{4} - \frac{\kappa_r^2}{4}]\cdot||r||^2 - \frac{\rho}{2}||\tilde{W}||_F^2 + \frac{\epsilon_N^2}{\kappa_r^2} + \frac{\rho}{2}W_B \tag{33}$$

Knowing the fact that $\tanh^2(||x||) \leq ||\mathrm{Tanh}(x)||^2$ *for all $x \in \mathbb{R}^n$, $\dot{V}$ can be further bounded as follows:*

$$\dot{V} \leq -\psi_3(||h||) + \beta_1 \tag{34}$$

where $\beta_1 = \frac{\epsilon_N^2}{\kappa_r^2} + \frac{\rho}{2}W_B$ and $\psi_3(||h||)$ is defined as $\psi_3(||h||) = \beta_2\tanh^2(||h||)$ and $\beta_2 \in \mathbb{R}$ is given by:

$$\beta_2 = \min\{\frac{\alpha}{1+\beta'} - 1, \underline{m}\lambda_{min}(K_1) - \frac{1+\kappa_r^2}{4}, \lambda_{min}(K_2), \frac{\rho}{\lambda_{max}(\varGamma^{-1})}\}. \tag{35}$$

The positiveness of β_2 is guaranteed by conditions (28)–(30), thus the closed-loop system is ISS. Furthermore, according to (10)*, $\dot{e}$ asymptotically converges to zero as h do.* ■

Thanks to the boundedness of $\hat{f}_1$, $\hat{f}_2$ and NN term, the control law (26) provides an a priori bounded control command when the signal χ is saturated. However, Theorem 4.1 only guarantees the convergence of the tracking error. To this end, a sufficient condition for K_2 is derived next, such that χ can be replaced by the following saturated function

$$\chi = \mathrm{Tanh}(s) \tag{36}$$

where the auxiliary variable $s = [s_1 \cdots s_n]^T \in \mathbb{R}^n$ is the solution of the following differential equation:

$$\dot{s} = \mathrm{Cosh}^2(s)[-K_1 r - K_2\mathrm{Tanh}(s) + \mathrm{Tanh}(e)] \tag{37}$$

in which $s(0) = 0$, and the matrix $\mathrm{Cosh}(\cdot) \in \mathbb{R}^{n\times n}$ is defined in (15). The satisfaction of (36) will impose the following bound on χ:

$$||\chi|| \leq \sqrt{n} \tag{38}$$

where n denotes the dimension of χ.

According to (37), the time derivative of $\mathrm{Tanh}(s)$ can be obtained as:

$$\frac{d}{dt}\mathrm{Tanh}(s) = -K_1 r - K_2 \mathrm{Tanh}(s) + \mathrm{Tanh}(e) \quad (39)$$

Now, subtracting (39) from both sides of (11) yields

$$\dot{\chi} - \frac{d}{dt}\mathrm{Tanh}(s) = -K_2[\chi - \mathrm{Tanh}(s)] \quad (40)$$

with $\chi(0) = \mathrm{Tanh}(s(0)) = 0$. Therefore, from (40) one can conclude that $\chi(t) = \mathrm{Tanh}(s(t))$ provided s is bounded. So the following condition needs to be verified by K_2:

$$\begin{aligned}\lambda_{min}(K_2) &\geq [1 + \lambda_{max}(K_1) \cdot (\beta' + \alpha + 1)] \cdot \sqrt{n} \quad (41)\\ &\geq ||\mathrm{Tanh}(e)|| + \lambda_{max}(K_1) \cdot ||r||\end{aligned}$$

where $(\beta'+\alpha+1)\sqrt{n} \geq ||r||$ represents the upper bound of r accroding to (10). Hence, if parameters K_1, K_2 and α satisfy the condition (41), the boundedness of χ is guaranteed and consequently, τ is bounded as well. Next, we show that, via proper selection of the tuning parameters, the input signal τ can be further rendered to verify the inequality defined in Problem 1.

Lemma 1 *Suppose Assumptions 1-2 hold and the ISS stability of closed-loop system is verified, then the control input τ given by (26) is guaranteed to be bounded by the τ_N, i.e. $||\tau|| \leq \tau_N$, if the tunning parameters α, K_1, K_2, and bounds of desired trajectory B_v, B_a satisfy* (41) *and the following inequalities*

$$\lambda_{max}(K_1) - \lambda_{min}(K_1) < \frac{\tau_N - \tau_m}{2(1+\beta')\overline{m}n} \quad (42)$$

$$g_1(\alpha, K_1, K_2) + g_2(B_v, B_a) < \tau_N \quad (43)$$

where τ_m defined in (52) *is a positive constant, g_1 defined in* (50) *and g_2 defined in* (51) *are positive constants depends on the value of tuning parameters. Moreover, η is then set as $\eta = 1 - \frac{g_1(\alpha,K_1,K_2)+g_2(B_v,B_a)}{\tau_N}$.*

Proof 2 *Start with the NN term of control law* (26)*, due to* (23) *and the projection operator* (25)*, one can easily verify*

$$||\Delta\hat{f}|| \leq \eta \cdot \tau_N \quad (44)$$

Secondly, by (38)*, the upper bound of the first term of control law* (26) *can be written as:*

$$||K_1\chi|| \leq ||K_1|| \cdot \sqrt{n}. \quad (45)$$

As for $\hat{f}_1$ and $\hat{f}_2$, according to Property 2 and Assumption 1, it holds that

$$||\hat{f}_1|| \leq \overline{m}[\alpha n + ||K_2|| \cdot \sqrt{n} + \sqrt{n}] \quad (46)$$

$$\begin{aligned}||\hat{f}_2|| \leq &\zeta_c B_v\left[B_v + \frac{\alpha}{1+\beta'}\sqrt{n} + \sqrt{n}\right] + \zeta_{f_d} B_v\\ &+ \overline{m}B_a + \zeta_{f_s} + \zeta_g \quad (47)\end{aligned}$$

where $\hat{f}_1$ and $\hat{f}_2$ defined in (16) and (17), respectively. Applying Young's inequality to term $\frac{\zeta_c\sqrt{n}}{1+\beta'}B_v\alpha$ of (47)*, we get*

$$\frac{\zeta_c\sqrt{n}}{1+\beta'}B_v\alpha \leq B_v^2\frac{\zeta_c^2}{4\overline{m}} + \frac{\alpha^2}{(1+\beta')^2}\overline{m}n \quad (48)$$

Thanks to (45)-(48) *and due to the fact that $\sqrt{n} \leq n$, we can bound the first three terms of* (26) *by:*

$$||K_1\chi|| + ||\hat{f}_1|| + ||\hat{f}_2|| \leq g_1 + g_2 \quad (49)$$

where g_1 and g_2 are defined as follows:

$$g_1 = ||K_1||n + ||K_2||\overline{m}\sqrt{n} + \left[\frac{\alpha^2}{(1+\beta')^2} + \alpha + 1\right]\overline{m}n, \quad (50)$$

$$g_2 = B_v^2(\frac{\zeta_c^2}{4\overline{m}} + \zeta_c) + B_v(\zeta_c n + \zeta_{f_d}) + \zeta_{f_s} + \zeta_g + B_a\overline{m}. \quad (51)$$

Considering (28)−(30) *and* (41)*, we can obtain that the lower bound of g_1 and g_2, denoted by $\underline{g_1}$ and $\underline{g_2}$, as*

$$\begin{aligned}\underline{g_1} &= \frac{(1+\kappa_r^2)n}{4\underline{m}} + [4+\beta'+2\lambda_{max}(K_1)(1+\beta')]\overline{m}n\\ &\geq \frac{(1+\kappa_r^2)n}{4\underline{m}} + [4+\beta'+2\lambda_{min}(K_1)(1+\beta')]\overline{m}n\\ &= \frac{(1+\kappa_r^2)n}{4\underline{m}} + [4+\beta'+\frac{1+\kappa_r^2}{2\underline{m}}(1+\beta')]\overline{m}n := \tau_m \quad (52)\\ \underline{g_2} &= \zeta_{f_s} + \zeta_g\end{aligned}$$

Thanks to Assumption 2 and if condition (42) *is satisfied, we have $\underline{g_1} + \underline{g_2} < \tau_N$ such that there always exist α, K_1, K_2, B_v and B_a that verify condition* (43)*, and consequently guarantee $g_1 + g_2 < \tau_N$.*

Then , according to (49)*, we have*

$$||K_1\chi|| + ||\hat{f}_1|| + ||\hat{f}_2|| < \tau_N. \quad (53)$$

In summary, given the selection of η in Lemma 1, (44) *and* (53)*, the output of actuator is ensured to be bounded by*

$$||\tau|| \leq ||K_1\chi|| + ||\hat{f}_1|| + ||\hat{f}_2|| + ||\Delta\hat{f}|| \leq \tau_N \quad (54)$$

Thus ending the proof. ■

Remark 4.1 *It can be seen that the greater the uncertainty of the model, the smaller the feasible range of B_v and B_a, in the case of a fixed τ_N. However, by tuning η, we can flexible allocate the control efforts to either achieve fast tracking or handle the uncertainty.*

5 Numerical Experiment

In this section, a two-link robot is used to demonstrate the effectiveness of the proposed controller. The matrices defined in (1) are same as [35, Section III] except for the friction vector which given by $F(\dot{q}) = 0.5\dot{q}$. The desired tracking trajectories of the manipulator are chosen as

$$q_d = [\sin(t) \quad \cos(t)]^T$$

The initial conditions of system are $q(0) = [0.5, 0.5]^T$, $\dot{q}(0) = [0, 0]^T$. The RBF NN contains 30 neurons with $p_{ik} = 1.5$, μ_i evenly located in $[-3 \quad 3]$ and $\hat{W}(0) = \mathbf{0}_{30\times 2}$. The tunning parameters of the proposed control algorithm are set as $\Gamma = \mathrm{diag}\{30, \cdots, 30\}$, $\rho = 0.001$, $\beta = 1.5$, $\alpha = 4$, $K_1 = \mathrm{diag}\{15, 15\}$, $K_2 = \mathrm{diag}\{200, 200\}$. The external disturbance terms are modeled by

$$T_d(t) = [0.5 + 0.5\sin(0.1t) \quad 0.5 + 0.5\cos(0.1t)]^T$$

The proposed scheme is compared with a standard proportional-differential (PD) controller and a state-of-art feedback neural network (FNN) controller [21].

Case 1: In first case, input constraint and model uncertainty are not included, i.e. τ_N assumed to be infinitely large and the nominal model is accurate.

For the fairness of comparison, we first tune all three methods to achieve similar convergence speed, resulting in parameters of PD and FNN methods are: 1) PD controller, $K_p = \text{diag}\{1022, 1022\}$, $K_d = \text{diag}\{200, 200\}$; 2) FNN controller, $\alpha = 4$, $K_1 = \text{diag}\{380, 380\}$, $K_2 = \text{diag}\{500, 500\}$, $\mathit{\Gamma} = \text{diag}\{30, \cdots, 30\}$, $\rho = 0.001$, the neural network is set to be same with our controller.

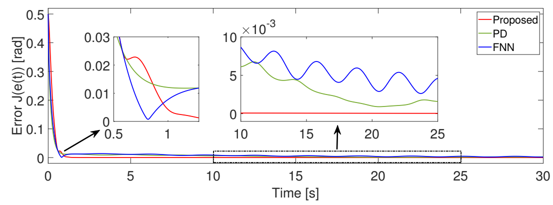

Fig. 1: Time evolution of the trajectories error in case 1.

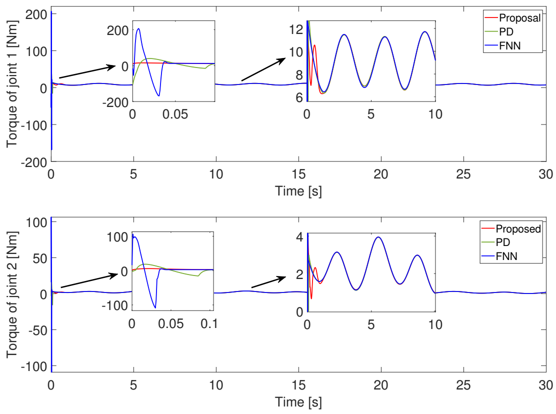

Fig. 2: Time evolution of the input torque of joint without limitation.

As shown in Fig.1, three methods are able to track q_d with sufficiently small tracking error after around 1s. The convergence criteria shown is selected as the root mean square error (RMSE) of tracking trajectories, defined as

$$J(e(t)) = \sqrt{\frac{1}{2}(e_1^2 + e_2^2)} \tag{55}$$

where e_i, $i = 1, 2$, denotes the tracking error of joint i. With similar convergence speed, the controller proposed demonstrates obviously a smaller stead state error. The input torque is depicted in Fig.2. Noted that, both PD controller and FNN controller, have a large overshoot during the transient period while the proposed method features a smooth trajectory that kept in a small range. This proves the effectiveness of the proposed controller.

Case 2: Next, we take the model uncertainty and input constraint into account. The limits of torque of actuator output are set as $\tau_N = 15$Nm. The percentage of the model mismatch resulting from inaccurate information of system parameters is set to 50%. Since there is model uncertainty, η of the proposed method is now set to 0.8. Simulation results are shown in Fig. 3-4.

Fig.3 shows the time history of trajectory tracking errors. Compared with Fig.1, it can be seen that, PD controller loses the ability of tracking the given trajectory while the performance of the FNN method are significantly degraded in terms of steady state error. Although the proposed method also loses some tracking accuracy in this case, the input signal of actuator, as shown in Fig. 4, features a smooth and bounded trajectory. It is worth pointing out that the proposed control law indeed does not to exceed the limitation of actuator, which verifies the analysis in Lemma 1. However, as shown in Fig.4, the FNN method suffers a lot form the input constraint. Although the boundedness of the control signal is proved in [21], since the boundary is non-adjustable, the performance of the controller still decline in case of small τ_N Moreover, the switching-like behaviour of the torque in this example would certainly damage the actuator and waste the energy. The behaviour of torque of joint 2 is similar to joint 1, therefore is omitted here due to the limit of space.

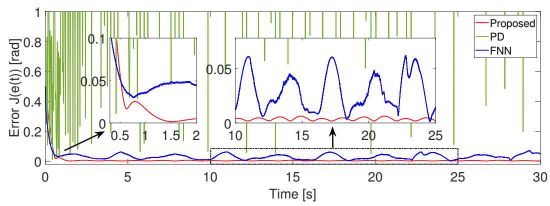

Fig. 3: Time evolution of the trajectories error in case 2.

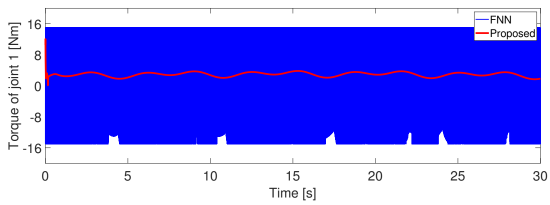

Fig. 4: Time evolution of the input torque of joint 1 with limitation.

6 Concluding Remarks

In this paper, a novel adaptive NN control law is proposed to solve tracking problem of robot system with model uncertainty and external disturbance. The proposed scheme generates a bounded output of actuator by utilizing a projection operation and an auxiliary term admits the $\tanh(\cdot)$ form. The major novelty of this paper is that the output signals of controller are always guaranteed to be bounded and not to exceed the limits of actuator. Compared with the existing controllers by using simulation, we confirm the effectiveness and superiority of the proposed adaptive NN controller in dealing with model uncertainty and input constraint of actuator.

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